We consider the problem related to the existence of continuous periodic solutions with two switching points fixed in phase space and the period multiple to the period of the function describing external influence. A switching point of a solution to the system in phase space is said to be a state of the system such that the nonlinear function takes one of its threshold numbers and changes an output number, i.e. the switch occurs in a relay. The main result of this research work is the following theorem. The proof of the theorem is found in [2].

Theorem. *Let the following conditions hold:*

1) *the external influence of the system is a T-periodic function containing the sum of two sine functions and a constant;*

2) *the virtual points of stability in phase space of the system lie out of the non-single-valued zone of the function describing hysteresis nonlinearity;*

3) *the initial system is reduced to the special canonical form by the nonsingular transformation if the system is completely controllable with respect to the nonlinearity and the eigenvalues of the system matrix are real, prime, and nonzero;*

4) *the coefficients of the real vector defining feedback in the canonical system are nonzero except for one, which is zero;*

5) *the switching instants and the switching points of the image point of the solution to the canonical system are the solutions to the auxiliary system of transcendental equations constructing of which is based on the assumption that there exists at least one periodic solution with two switching points and which parameters satisfy the conditions of its solvability.*

Then, for given $k \in \mathbb{N}$, there exists a unique kT-periodic solution to the initial system with two switching points belonging to the switching hyperplanes, where the switching points can be calculated with the inverce transformation.

Remark. The second switching instant equals the period T provided the image point begins its motion at $t = 0$.

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POLE ASSIGNMENT IN DISCRETE-TIME LINEAR SYSTEMS WITH INCOMPLETE STATE FEEDBACK

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Consider a dicrete-time linear control system

$$x(t+1) = A(t)x(t) + B(t)u(t), \quad y(t) = C^*(t)x(t), \quad t \in \mathbb{Z}, \quad (x,u,y) \in \mathbb{K}^{n+m+k}, \tag{1}$$

with a linear incomplete feedback $u(t) = U(t)y(t)$, where $\mathbb{K} = \mathbb{C}$ or $\mathbb{K} = \mathbb{R}$. By $X(t,s)$ denote the Cauchy matrix of unforced system $x(t+1) = A(t)x(t)$. System (1) is called *consistent on an interval* $[t_0, t_1)$ if for any $n \times n$-matrix $G \in M_n$ there exists a feedback gain $\widehat{U}(t)$, $t \in [t_0, t_1)$, that transfers the solution of the $n \times n$-matrix system

$$Z(t+1) = A(t)Z(t) + B(t)\widehat{U}(t)C^*(t)X(t,t_0), \quad t \in \mathbb{Z},$$

from the state $Z(t_0) = 0$ to the state $Z(t_1) = G$. Suppose that $(A(t), B(t), C(t)) \equiv \equiv (A, B, C)$ and $U(t) \equiv U$. Denote by $\chi(\lambda; U) = \det(\lambda I - A - BUC^*)$ the characteristic polynomial of the closed-loop system

$$x(t+1) = (A + BUC^*)x(t), \quad t \in \mathbb{Z}, \quad x \in \mathbb{K}^n. \tag{2}$$

We say that the system (2) is *arbitrarily pole assignable* if for any monic polynomial $p(\lambda) = = \lambda^n + \gamma_1 \lambda^{n-1} + \ldots + \gamma_n$ with $\gamma_i \in \mathbb{K}$ there exists a $m \times k$-matrix gain U over the field $\mathbb{K}$ such that $\chi(\lambda; U) = p(\lambda)$.

Suppose that the coefficients of the system (2) have the following form:

$$A = \{a_{ij}\}_{i,j=1}^n, \quad a_{i,i+1} \neq 0, \quad i = \overline{1, n-1}; \quad a_{ij} = 0, \quad j > i+1; \tag{3}$$

$$B = \{b_{ij}\}, \quad C = \{c_{is}\}, \quad i = \overline{1, n}, \quad j = \overline{1, m}, \quad s = \overline{1, k}; \tag{4}$$

$$b_{ij} = 0, \quad i = \overline{1, p-1}, \quad j = \overline{1, m}; \quad c_{is} = 0, \quad i = \overline{p+1, n}, \quad s = \overline{1, k}; \quad p \in \{\overline{1, n}\}. \tag{5}$$

Theorem 1. *Let the coefficients of the system* (2) *have the form* (3), (4), (5). *Then the implications* $1 \Longrightarrow 2 \Longleftrightarrow 3$ *hold for the following conditions.*

1. *System* (2) *is consistent.*
2. *The matrices* C^*B, C^*AB, $\ldots$, $C^*A^{n-1}B$ *are linearly independent.*
3. *System* (2) *is arbitrarily pole assignable.*

Theorem 2. *Let the coefficients of the system* (2) *have the form* (3), (4), (5), *and* $\det A \neq 0$. *Then the implication* $2 \Longrightarrow 1$ *in Theorem* 1 *holds if at least one of the following conditions is satisfied:*

a) $\operatorname{rank} B = n$;
b) $\operatorname{rank} C = n$;
c) *all eigenvalues of matrix* A *are equal*;
d) $n < 6$.

The above results are similar to the corresponding results [1] for continuous-time systems.

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